

E C O N O M I C S B U L L E T I N

An irrelevance result with differentiated goods

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Abstract

White (1996), Poyago-Theotoky (2001) and Myles (2002) prove that in the mixed oligopoly the optimal subsidy, equilibrium output level, all firms' profits and social welfare are identical irrespective of whether the public firm maximizes welfare or profit and moves simultaneously with private firms, or maximizes welfare and acts as a Stackelberg leader. They name this observation the 'irrelevance result'. Previous results have assumed all firms produce a homogeneous product with quantity as the strategic variable. We show that the irrelevance result extends to product differentiation and to Bertrand competition with price as the strategic variable.

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1 Introduction

In recent years mixed oligopoly models have received a growing interest in economic theory. These models assume that the public firm maximizes social welfare, defined as the sum of consumer surplus and firm profits minus subsidy, while private firms maximize profit. The main focus has been the consequences of privatisation of the public firm. Early studies, e.g. DeFraja and Delbono (1989), show that privatisation of the public firm is desirable in terms of social welfare. On the other hand, White (1996) proves that this result is not always true. In an oligopolistic framework, White (1996) showed that, in the case of government intervention in the mixed oligopoly market by using an optimal output subsidy, privatisation of the public firm is ineffective because under the optimal output subsidy, firms' output and profit and social welfare are identical both before and after privatisation of the public firm. Poyago-Theotoky (2001) shows that the optimal output subsidy is identical and profits, output and social welfare are also identical irrespective of whether (i) the public firm moves simultaneously with private firms or (ii) it acts as a Stackelberg leader or (iii) all firms, public and private, behave as profit-maximizers. Myles (2002) extends the irrelevance result of Poyago-Theotoky from linear inverse demand to general inverse demand and cost functions.

So far almost all of the literature has assumed Cournot (quantity) competition with a homogeneous product. It is hence important to consider whether the conclusions drawn remain valid under alternative assumptions. Our main conclusion is that the irrelevance result applies even with product differentiation, and also extends to Bertrand (price) competition. It is therefore significantly more general than previously suspected.

This paper is organized as follows. Section 2 presents a general framework of a mixed oligopoly market for differentiated products and Cournot competition and Section 3 examines the analysis for Bertrand competition. Section 4 concludes the paper.

2 Cournot Competition With Differentiated Goods

Differentiated goods, labeled by $i = 0, 1, \dots, n$, are each produced by one firm. Firm 0 is a public firm, and the rest are private. We consider the social

optimum and the market equilibrium outcomes before and after privatisation of the public firm. There is one consumer. The inverse demands and the direct demands are:

$$\begin{aligned} p_i &= p_i(q_0, q_1, \dots, q_n), & \frac{\partial p_i}{\partial q_i} < 0, & \quad \frac{\partial p_i}{\partial q_j} \neq 0, \\ q_i &= q_i(p_0, p_1, \dots, p_n), & \frac{\partial q_i}{\partial p_i} < 0, & \quad \frac{\partial q_i}{\partial p_j} \neq 0. \end{aligned}$$

Note that these demand functions permit homogeneous products as a special case. In the previous literature it was assumed that the firms produce a homogeneous good, and the welfare measure was defined as the sum of consumer surplus and firms' profits less subsidy costs. Note that with many differentiated goods consumer surplus is not generally well-defined. If there is no cross-price effect in the demand functions one can arguably use the sum of consumer surpluses for individual goods as the total consumer surplus. This, however, is not an appropriate welfare measure when cross-price effects are present. Instead, the indirect utility function can be used.

The consumer owns the private firms. The consumer, as a government, owns the public firm and pays out subsidies to the firms. Total income is defined as a lump-sum income plus total profits less total subsidies

$$w = M + \sum \pi_i - \sum s_i q_i,$$

where the profits are

$$\pi_i = p_i q_i - C_i(q_i) + s_i q_i.$$

The indirect utility function is

$$\begin{aligned} V(\mathbf{p}, w) &= V\left(\mathbf{p}, M + \sum [p_i q_i - C_i(q_i) + s_i q_i] - \sum s_i q_i\right) \\ &= V\left(\mathbf{p}, M + \sum [p_i q_i - C_i(q_i)]\right). \end{aligned}$$

We assume that this function is differentiable.

In the social optimum the quantities are chosen to maximize the indirect utility. The first-order conditions are

$$\sum_{j=0}^n \frac{\partial V}{\partial p_j} \frac{\partial p_j}{\partial q_i} + \frac{\partial V}{\partial w} \left[p_i + \sum_{j=0}^n \left(q_j \frac{\partial p_j}{\partial q_i} \right) - C'_i(q_i) \right] = 0, \quad i = 0, \dots, n, \quad (1)$$

assuming an interior solution. Using Roy's identity for the equilibrium,

$$\frac{\partial V(\mathbf{p}^*, w^*)}{\partial p_j} = -q_j^* \frac{\partial V(\mathbf{p}^*, w^*)}{\partial w},$$

the set of equations defining the socially-optimal equilibrium is simplified to

$$p_i^* = C'_i(q_i^*), \quad i = 0, \dots, n. \quad (2)$$

It is assumed that this set of equations has a unique solution.

2.1 Mixed and Private Cournot-Nash Oligopoly

In the mixed Cournot oligopoly firm 0 chooses q_0 to maximize indirect utility, and firms $i = 1, \dots, n$ choose q_i to maximize their profit. The first-order condition for the public firm is

$$\sum_{j=0}^n \frac{\partial V}{\partial p_j} \frac{\partial p_j}{\partial q_0} + \frac{\partial V}{\partial w} \left[p_0 + \sum_{j=0}^n \left(q_j \frac{\partial p_j}{\partial q_0} \right) - C'_0(q_0) \right] = 0,$$

and is the same as in (1). Hence, using Roy's identity,

$$p_0^m = C'_0(q_0^m).$$

For private firm i the first-order condition is

$$0 = \frac{\partial \pi_i}{\partial q_i} = p_i + q_i \frac{\partial p_i}{\partial q_i} - C'_i(q_i) + s_i, \quad i = 1, \dots, n. \quad (3)$$

Setting the subsidy equal to s_i^{mC} defined by

$$s_i^{mC} = -q_i^* \frac{\partial p_i(\mathbf{q}^*)}{\partial q_i}, \quad i = 1, \dots, n, \quad (4)$$

results in the socially-optimal outcome (2).

After privatisation all firms maximize their profits by choosing outputs. The socially optimal outcome is achieved by setting

$$s_0 = -q_0^* \frac{\partial p_0(\mathbf{q}^*)}{\partial q_0},$$

with subsidies s_i^{mC} as defined in (4) to firms $i = 1, \dots, n$.

2.2 Mixed Cournot-Stackelberg Oligopoly

If the public firm is Stackelberg leader in the mixed oligopoly then it solves

$$\begin{aligned} & \sum_{j=0}^n \frac{\partial V}{\partial p_j} \left[\frac{\partial p_j}{\partial q_0} + \sum_{k=1}^n \frac{\partial p_j}{\partial q_k} \frac{\partial q_k}{\partial q_0} \right] \\ & + \frac{\partial V}{\partial w} \left[p_0 + \sum_{j=1}^n p_j \frac{\partial q_j}{\partial q_0} + \sum_{j=0}^n q_j \left[\frac{\partial p_j}{\partial q_0} + \sum_{k=1}^n \frac{\partial p_j}{\partial q_k} \frac{\partial q_k}{\partial q_0} \right] \right. \\ & \left. - C'_0(q_0) - \sum_{k=1}^n \left(C'_k(q_k) \frac{\partial q_k}{\partial q_0} \right) \right] = 0, \end{aligned}$$

where firm k 's reaction function $\frac{\partial q_k}{\partial q_0}$ satisfies (3). Collecting the terms,

$$\begin{aligned} & \sum_{j=0}^n \frac{\partial V}{\partial p_j} \frac{\partial p_j}{\partial q_0} + \frac{\partial V}{\partial w} \left[p_0 + \sum_{j=0}^n \left(q_j \frac{\partial p_j}{\partial q_0} \right) - C'_0(q_0) \right] \\ & + \sum_{k=1}^n \left(\sum_{j=1}^n \left[\left(\frac{\partial V}{\partial p_j} + \frac{\partial V}{\partial w} q_j \right) \frac{\partial p_j}{\partial q_k} \right] + \frac{\partial V}{\partial w} [p_k - C'_k(q_k)] \frac{\partial q_k}{\partial q_0} \right) = 0, \end{aligned}$$

and using again the Roy's identity for the equilibrium we obtain

$$p_0^* - C'_0(q_0^*) + \frac{\partial V(\mathbf{p}^*, w^*)}{\partial w} \sum_{k=1}^n [p_k^* - C'_k(q_k^*)] \frac{\partial q_k}{\partial q_0} \Big|_{q_0^*} = 0$$

This results in the socially-optimal outcome if the optimal subsidy (4) is given to the private firms.

Hence we have proved that the irrelevance result holds with differentiated goods for Cournot-Nash mixed and private duopoly cases, and it also holds when the public firm is a leader in Cournot-Stackelberg duopoly.

3 Bertrand Competition With Differentiated Goods

The socially-optimal equilibrium is the same regardless of the choice variables. Solving the first-order conditions with respect to prices,

$$\begin{aligned} 0 &= \frac{\partial}{\partial p_i} V \left(\mathbf{p}, M + \sum_{j=0}^n [p_j q_j - C_j(q_j)] \right) \\ &= \frac{\partial V}{\partial p_i} + \frac{\partial V}{\partial w} \left[q_i + \sum_{j=0}^n \left(p_j \frac{\partial q_j}{\partial p_i} - C'_j(q_j) \frac{\partial q_j}{\partial p_i} \right) \right], \quad i = 0, \dots, n, \end{aligned}$$

and using Roy's identity for the equilibrium we obtain

$$\sum_{j=0}^n \left(p_j^* - C'_j(q_j^*) \right) \frac{\partial q_j(\mathbf{p}^*)}{\partial p_i}, \quad i = 0, \dots, n,$$

which is the same as (2) given the assumptions on the direct demand functions.

3.1 Mixed and Private Bertrand-Nash Oligopoly

In the mixed Bertrand oligopoly firm 0 chooses p_0 to maximize indirect utility, and firms $i = 1, \dots, n$ choose p_i to maximize their profit. The first-order condition for the public firm is

$$\frac{\partial V}{\partial p_0} + \frac{\partial V}{\partial w} \left[q_0 + \sum_{j=0}^n \left(p_j \frac{\partial q_j}{\partial p_0} - C'_j(q_j) \frac{\partial q_j}{\partial p_0} \right) \right] = 0, \quad (5)$$

and for the private firms

$$q_i + p_i \frac{\partial q_i}{\partial p_i} - C'_i(q_i) \frac{\partial q_i}{\partial p_i} + s_i \frac{\partial q_i}{\partial p_i} = 0, \quad i = 1, \dots, n.$$

Setting the subsidies equal to s_i^{mB} defined as

$$s_i^{mB} = - \frac{q_i^*}{\left(\frac{\partial q_i(\mathbf{p}^*)}{\partial p_i} \right)}, \quad i = 1, \dots, n, \quad (6)$$

and using Roy's identity in (5) for the equilibrium we obtain (2).

After privatisation all firms maximize their profits by choosing outputs, and so the socially optimal outcome is achieved by setting

$$s_0 = -\frac{q_0^*}{\left(\frac{\partial q_0(\mathbf{p}^*)}{\partial p_0}\right)},$$

with subsidies s_i^{mB} as defined in (6) to firms $i = 1, \dots, n$.

3.2 Mixed Bertrand-Stackelberg Oligopoly

If the public firm is the Stackelberg leader in the mixed oligopoly then it solves

$$\begin{aligned} & \frac{\partial V}{\partial p_0} + \sum_{j=1}^n \frac{\partial V}{\partial p_j} \frac{\partial p_j}{\partial p_0} \\ & + \frac{\partial V}{\partial w} \left[q_0 + \sum_{j=1}^n q_j \frac{\partial p_j}{\partial p_0} + \sum_{j=0}^n p_j \left(\frac{\partial q_j}{\partial p_0} + \sum_{k=1}^n \frac{\partial q_j}{\partial p_k} \frac{\partial p_k}{\partial p_0} \right) \right. \\ & \left. - \sum_{j=0}^n C'_j(q_j) \left(\frac{\partial q_j}{\partial p_0} + \sum_{k=1}^n \frac{\partial q_j}{\partial p_k} \frac{\partial p_k}{\partial p_0} \right) \right] = 0, \end{aligned}$$

where $p_j(p_0)$ is firm j 's reaction function. Collecting the terms,

$$\begin{aligned} & \frac{\partial V}{\partial p_0} + \frac{\partial V}{\partial w} \left[q_0 + \sum_{j=0}^n [p_j - C'_j(q_j)] \frac{\partial q_j}{\partial p_0} \right] \\ & + \sum_{k=1}^n \frac{\partial p_k}{\partial p_0} \left[\frac{\partial V}{\partial p_k} + \frac{\partial V}{\partial w} \left(q_k + \sum_{j=0}^n [p_j - C'_j(q_j)] \frac{\partial q_j}{\partial p_k} \right) \right] = 0, \end{aligned}$$

and using Roy's identity for the equilibrium, with subsidies (6) we obtain the socially-optimal outcome.

Observe that the subsidies in Cournot and Bertrand cases are different, since in general

$$\frac{\partial p_i(\mathbf{q}^*)}{\partial q_i} \neq \frac{1}{\left(\frac{\partial q_i(\mathbf{p}^*)}{\partial p_i}\right)}.$$

4 Conclusion

This paper has demonstrated that under the optimal subsidy, all firms' profits, prices, output and welfare are identical regardless of the nature of the competition. Our finding has been obtained for the general demand and cost functions, and for an arbitrary number of differentiated goods. The irrelevance result therefore extends well beyond the original setting of quantity competition with homogeneous products.

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